Review for Final

Unit 1: Algebra Foundations

The Real Number System

THE REAL NUMBERS (R): All Numbers!

IRRATIONAL NUMBERS (I): $\pi, \sqrt{10}$ ← crazy Numbers!

RATIONAL NUMBERS (Q): $3/2, 0.\overline{6}, -4.5$

INTEGERS (Z): $\ldots -3, -2, -1, 0, 1, 2, 3 \ldots$

WHOLE NUMBERS (W): $0, 1, 2, 3 \ldots$

NATURAL NUMBERS (N): $1, 2, 3, 4 \ldots$

Name all sets to which each number belongs

1. $\frac{2}{3}$ R, Q
2. 13 R, Q, Z, W, N
3. 0 R, Q, Z, W
4. $-\sqrt{50}$ R, I
5. $-\frac{28}{7}$ R, Q, Z
6. π R, I

-4

Properties

COMMUTATIVE: ORDER!! • $a+b=b+a$ • $ab=ba$	**ASSOCIATIVE:** Grouping! • $(a+b)+c=a+(b+c)$ • $(ab)c=a(bc)$
IDENTITY: • $a+\boxed{0}=a$ • $a\cdot\boxed{1}=a$	**INVERSE:** • $a+\boxed{-a}=0$ • $a\cdot\boxed{\frac{1}{a}}=1$ (reciprocal)
ZERO PRODUCT: • $a\cdot 0=0$ • $(x^2+1)\cdot 0=0$	**DISTRIBUTIVE:** • $a(b+c)=ab+ac$ • $-7(x-3)=-7x+21$
REFLEXIVE: • $a=a$ • $17y=17y$	**SYMMETRIC:** • If $a=b$, then $b=a$ • If $3x=15$, then $15=3x$

TRANSITIVE:

- If $a=b$ and $b=c$, then $a=c$
- If $7=\sqrt{49}$ and $\sqrt{49}=6+1$, then $7=6+1$

Identify the following properties:

1. $5x+1=1+5x$ Commutative
2. $17=17$ Reflexive
3. $10y^2 \cdot 0=0$ Zero Product
4. $-3(x+8)=-3x-24$ Distributive
5. If $2^5=32$ and $32=8\cdot 4$, then $2^5=8\cdot 4$ Transitive
6. $8k+0=8k$ Identity
7. If $-2x=20$, then $20=-2x$ Symmetric
8. $\frac{4}{9}\cdot\frac{9}{4}=1$ Inverse

CLOSURE: *Answer yes or no. If no, give a counterexample.*

- Are **natural numbers** closed under **subtraction**? No $1-5=-4$
- Are **integers** closed under **addition**? Yes
- Are **irrational numbers** closed under **division**? No $\frac{\pi}{\pi}=1$
- Are **whole numbers** closed under **multiplication**? Yes

Square Roots & Cube Roots

1. $\sqrt{25}$ 5	2. $\sqrt{144}$ 12	3. $\sqrt{64}$ 8	4. $\sqrt{\frac{16}{49}}$ $\frac{\sqrt{16}}{\sqrt{49}} : \frac{4}{7}$

5. $\sqrt[3]{27}$ 3 3·3·3 9·3 27	6. $\sqrt[3]{216}$ 6 6·6·6 36·6 216	7. $\sqrt[3]{8}$ 2 2·2·2 4·2 8	8. $\sqrt[3]{1000}$ 10

Exponent Rules

PRODUCT RULE	POWER RULE	QUOTIENT RULE	NEGATIVE EXPONENT RULE
$x^a \cdot x^b = x^{a+b}$	$(x^a)^b = x^{ab}$	$\frac{x^a}{x^b} = x^{a-b}$	$x^{-a} = \frac{1}{x^a}$

1. $v^4 \cdot 7v^3 \cdot 5v$

$35v^8$

2. $(3x^2y^2)^3$

$27x^6y^6$

3. $(-2a^6bc^3)^2 \cdot -5ab^2$

$-20a^{13}b^4c^6$

4. $(-2y^4) \cdot (xy^3)^2 - 13x^2y^{10}$

$-15x^2y^{10}$

5. $\frac{a^6b^7c^2}{a^5b^4c^2}$

ab^3

6. $\frac{(-3x^6)^2}{5x^3 \cdot 3x^3} = \frac{9x^{12}}{15x^6}$

$\frac{3x^6}{5}$

7. $\left(\frac{4x^4y^2}{6xy}\right)^2 = \frac{4x^4y^2}{6xy} \cdot \frac{4x^4y^2}{6xy} = \frac{16x^8y^4}{36x^2y^2} = \frac{4x^6y^2}{9}$

8. $\frac{-9n^8}{27n^{10}} = -\frac{1}{3}n^{-2} = -\frac{1}{3n^2}$

9. $\frac{a^{12}b^{-3}}{(ab)^{-4}} = \frac{a^{12}b^{-3}}{a^{-4}b^{-4}} = a^{16}b^{1}$

$a^{16}b$

Unit 2: Expressions

P
E
M D
A S

Evaluating Expressions (Numerical & Algebraic)

1. $2^3 \cdot (9-2) + \frac{12}{4} - |-5|$

$2^3 \cdot 7 + \frac{12}{4} - |-5|$

$2^3 \cdot 7 + \frac{12}{4} - 5$

$8 \cdot 7 + 3 - 5$

$56 + 3 - 5$

$\boxed{54}$

2. $8 - [12 \div (\sqrt{49} - 1)] + 1$

$8 - [12 \div (7-1)] + 1$

$8 - [12 \div 6] + 1$

$8 - 2 + 1$

$\boxed{7}$

3. $\dfrac{5^3 - 42 \div 6}{\sqrt[3]{8}}$

$\frac{118}{2}$

59

$5^3 42 \div 6$

$125 - 42 \div 6$

$125 - 7$

118

$\sqrt[3]{8}$

2

4. $w^2 - 5xy$ *if x = -3, w = -2 and y = 1*

$(-2)^2 - 5(-3)(1)$

$4 - 5(-3)(1)$

$4 + 15(1)$

$4 + 15$

19

5. $\frac{7c^2 + 5}{4a - b}$ *if a = 1, b = -5 and c = -4*

$\frac{7(-4)^2+5}{4(1)-(-5)}$ → $7(16)+5 \rightarrow 117$

$4(1)-(-5) \rightarrow 4+5 \rightarrow 9$

$\frac{117}{9} = 13$

6. $2|y| - x^2$ *if x = 6 and y = -3*

$2|-3|-(6)^2$

$2(3)-(6)^2$

$2(3)-36$

$6-36$

-30

Translating Equations & Inequalities

let x = the #

1. The quotient of twice a number and 7 is 20. $\frac{2x}{7} = 20$	**2.** Five less than the product of a number and 3 is 14. $3x - 5 = 14$
3. Seven times the difference of x and 4 is -10. $7(x-4) = -10$	**4.** The product of a number and four increased by one is at least 7. $4x + 1 \geq 7$

Unit 3: Equations

Equations

1. $18 = 3 - 3a$

$-3 \quad -3$

$\frac{15}{-3} = \frac{-3a}{-3}$

$-5 = a$

$a = -5$

2. $4 - \frac{1}{2}n = -12$

$-4 \quad -4$

$\left(\frac{-2}{1}\right)\left(-\frac{1}{2}n\right) = -16\left(\frac{-2}{1}\right)$

$n = 32$

3. $\frac{3}{4}x + 17 = 23$

$-17 \quad -17$

$\left(\frac{4}{3}\right)\frac{3}{4}x = 6\left(\frac{4}{3}\right)$

$x = 8$

4. $9y - 4(y + 1) = 31$

$9y - 4y - 4 = 31$

$5y - 4 = 31$

$+4 \quad +4$

$\frac{5y}{5} = \frac{35}{5}$

$y = 7$

5. $-6(w-4)+8w=2(w+9)$

$-6w+24+8w=2w+18$

$2w+24=2w+18$

$-2w \quad -2w$

$24 \neq 18$

$\emptyset$

6. $3m-(7m+12)=2(m-3)$

$3m-7m-12=2m-6$

$-4m-12=2m-6$

$-2m \quad -2m$

$-6m-12=-6$

$+12 \quad +12$

$\frac{-6m}{-6}=\frac{6}{-6}$

$m=-1$

7. $2x - 2(4x - 3) = 6 - 6x$

$2x - 8x + 6 = 6 - 6x$

$-6x + 6 = 6 - 6x$

$+6x \quad +6x$

$6 = 6$ $\boxed{\infty}$

8. $\frac{7}{x-8} = \frac{3}{x}$

$7x = 3(x - 8)$

$7x = 3x - 24$

$-3x \quad -3x$

$\frac{4x}{4} = \frac{-24}{4}$

$\boxed{x = -6}$

9. Given $A = \frac{1}{2}bh$, solve for h

$$\frac{A}{b} = \frac{\frac{1}{2}bh}{b}$$

$$(2)\frac{A}{b} = (2)\frac{1}{2}h$$

$$\boxed{\frac{2A}{b} = h}$$

10. Given $K = \frac{mv^2}{2}$, solve for m

$$2 \cdot K = \frac{mv^2}{2} \cdot 2$$

$$\frac{2K}{v^2} = \frac{mv^2}{v^2}$$

$$\boxed{\frac{2K}{v^2} = m}$$

ABSOLUTE VALUE EQUATIONS

1. $|x| = 9$

$x = 9 \quad x = -9$

$x = \{-9, 9\}$

2. $|6c - 3| = 21$

$6c - 3 = 21$ $\quad$ $6c - 3 = -21$

$+3 \quad +3$ $\quad$ $+3 \quad +3$

$\frac{6c}{6} = \frac{24}{6}$ $\quad$ $\frac{6c}{6} = \frac{-18}{6}$

$c = 4$ $\quad$ $c = -3$

$c = \{-3, 4\}$

3. $|7m| + 4 = 25$

$-4 \quad -4$

$|7m| = 21$

$\frac{7m}{7} = \frac{21}{7} \qquad \frac{7m}{7} = \frac{-21}{7}$

$m = 3 \qquad m = -3$

$m = \{-3, 3\}$

4. $5|2n + 8| + 3 = 33$

$-3 \quad -3$

$\frac{5|2n+8|}{5} = \frac{30}{5}$

$|2n+8| = 6$

$2n+8 = 6 \qquad 2n+8 = -6$

$-8 \quad -8 \qquad -8 \quad -8$

$\frac{2n}{2} = \frac{-2}{2} \qquad \frac{2n}{2} = \frac{-14}{2}$

$n = -1 \qquad n = -7$

$n = \{-7, -1\}$

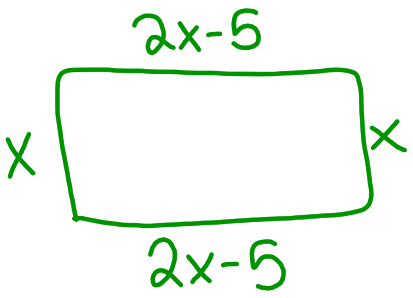
Unit 4: Word Problems

The length

1. Max is making a rectangular garden ~~that~~ is 5 feet less than twice its width. If the perimeter of the garden is 80 feet, what will be its dimensions?

let x = width

let 2x-5 = length



$80 = x + 2x - 5 + x + 2x - 5$

$80 = 6x - 10$

$+10 \quad +10$

$\frac{90}{6} = \frac{6x}{6}$

$15 = x$

$x = 15$

$2x - 5 = 25$

$2(15) - 5$

w = 15 ft

l = 25 ft

2. Amie published her first book. She was given $20,000 and an additional $0.15 for each copy of the book that sold. Her earnings, d, in dollars, from the publication of the book are given by **d = 20,000 + 0.15n**, where n is the number of copies sold. During the first year, Amie earned $22,100 from the publication and sale of her book. How many copies of her book were sold?

$d = 20{,}000 + 0.15n$

$22{,}100 = 20{,}000 + 0.15n$

$-20{,}000 \quad -20{,}000$

$\frac{2{,}100}{0.15} = \frac{0.15n}{0.15}$

$14{,}000 = n$

14,000 copies of her book were sold.

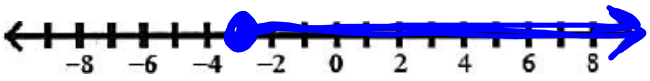
Unit 5: Inequalities

1. $11x + 13 \geq -20$

$-13 \quad -13$

$\frac{11x}{11} \geq \frac{-33}{11}$

$x \geq -3$



2. $-2x + 6 > 3x - 34$

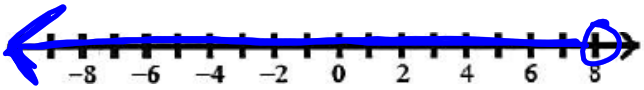
$-3x \quad -3x$

$-5x + 6 > -34$

$-6 \quad -6$

$\frac{-5x}{-5} > \frac{-40}{-5}$

$x < 8$



3. $3x - 7(x + 3) \geq -13$

$3x - 7x - 21 \geq -13$

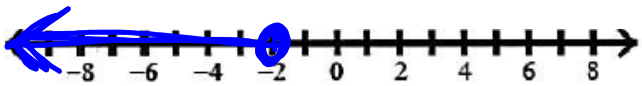
$-4x - 21 \geq -13$

$+21 \quad +21$

$-4x \geq 8$

$\frac{-4x}{-4} \geq \frac{8}{-4}$

$x \leq -2$



4. $4 - 8x < 2(5 - 3x)$

$4 - 8x < 10 - 6x$

$+6x \quad +6x$

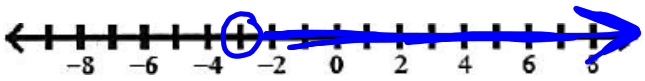
$4 - 2x < 10$

$-4 \quad -4$

$-2x < 6$

$\frac{-2x}{-2} > \frac{6}{-2}$

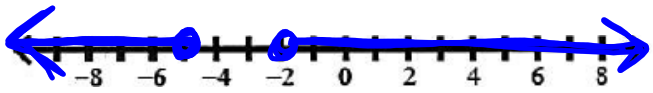
$x > -3$



5. $x + 7 \leq 2$ or $x + 5 \geq 3$

$-7 \quad -7 \qquad -5 \quad -5$

$x \leq -5$ or $x \geq -2$



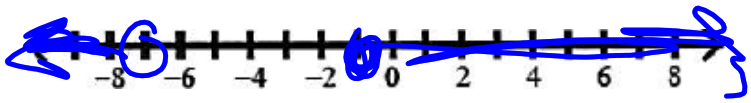
$(-\infty, -5]$ or $[-2, \infty)$

6. $3x + 5 < -16$ or $-5x - 8 \leq -13$

$-5 \quad -5 \qquad +8 \quad +8$

$\frac{3x}{3} < \frac{-21}{3}$ $\qquad$ $\frac{-5x}{-5} \leq \frac{-5}{-5}$

$x < -7$ $\qquad$ $x \leq -1$



$(-\infty, -7)$ or $[-1, \infty)$

7. $-2 \leq 3x - 2 < 10$

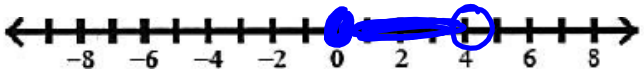
$+2 \quad +2 \quad +2$

$0 \leq 3x < 12$

$\frac{0}{3} \leq \frac{3x}{3} < \frac{12}{3}$

$0 \leq x < 4$

$[0, 4)$



8. $3 < 2x + 1 < 13$

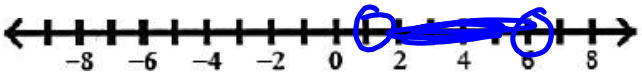
$-1 \quad -1 \quad -1$

$2 < 2x < 12$

$\frac{2}{2} < \frac{2x}{2} < \frac{12}{2}$

$1 < x < 6$

$(1, 6)$



Unit 6: Polynomials

Simplifying Polynomials

1. $(5 + 2x^3 + x - 3x^2) + (4x^3 + 11 - 6x + 7x^2)$

$5+2x^3+x-3x^2+4x^3+11-6x+7x^2$

$6x^3+4x^2-5x+16$

2. $(2x^2 + 3x + 2) - (x^2 - 4x - 1)$

$2x^2+3x+2-1x^2+4x+1$

x^2+7x+3

3. $3a^2b^3(2a^2 - 7ab + b^2)$

$6a^4b^3 - 21a^3b^4 + 3a^2b^5$

4. $(x + 4)(x + 9)$

$x^2 + 4x + 9x + 36$

$x^2 + 13x + 36$

5. $(2a + 5b)(a - 3b)$

$2a^2 - 6ab + 5ab - 15b^2$

$2a^2 - 1ab - 15b^2$

6. $(x + 8)(x - 8)$

$x^2 - 8x + 8x - 64$

$x^2 - 64$

7. $(2y - 1)^2$

$(2y - 1)(2y - 1)$

$4y^2 - 2y - 2y + 1$

$4y^2 - 4y + 1$

8. $\frac{18a^3b + 12a^2b^2 - 6ab}{6ab}$

$\frac{18a^3b}{6ab} + \frac{12a^2b^2}{6ab} - \frac{6ab}{6ab}$

$3a^2 + 2ab - 1$

9. $\frac{-24x^4 + 48x^3 - 8x^2}{8x^3}$

$\frac{-24x^4}{8x^3} + \frac{48x^3}{8x^3} - \frac{8x^2}{8x^3}$

$-3x + 6 - x^{-1}$

$-3x + 6 - \frac{1}{x}$

Unit 7: Factoring

Factoring

GCF	DIFFERNCE OF TWO PERFECT SQUARES	BASIC TRINOMIAL	BOX METHOD
*Always check first!	$a^2 - b^2$	$x^2 + bx + c$	$ax^2 + bx + c$

Polynomials that cannot be factored are called prime!

1. $21c - 12$

$3(7c-4)$

2. $x^2y + 8x$

$x(xy+8)$

3. $75a^2b^3c - 30ab^2$

$15ab^2(5abc-2)$

4. $4m^2 - 81n^2$

$(2m+9n)(2m-9n)$

5. $12x^2 - 12$

$12(x^2-1)$

$12(x+1)(x-1)$

6. $27b - 75b^3$

$3b(9-25b^2)$

$3b(3-5b)(3+5b)$

7. $p^2 - 13p + 30$ (A = 30, M = −13)

$(p-10)(p-3)$

8. $n^3 - 4n^2 - 60n$

$n(n^2 - 4n - 60)$ (A = −60, M = −4)

$n(n-10)(n+6)$

9. $5w^2 - 15w - 20$

$5(w^2-3w-4)$

$5(w+1)(w-4)$

10. $3x^2 + 10x + 3$

	3x	+1
x	$3x^2$	+1x
+3	+9x	+3

A	M
+10x	$+9x^2$

+1x

+9x

$(3x+1)(x+3)$

11. $12c^2 + 5c - 2$

	3c	+2
4c	$12c^2$	+8c
-1	-3c	-2

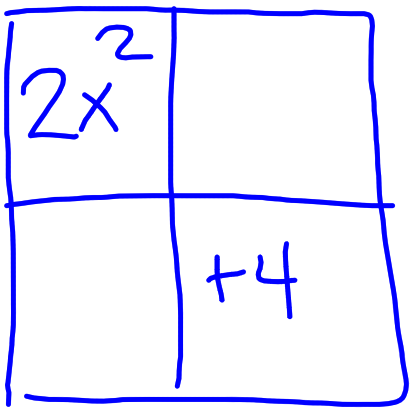
A	M
+5c	$-24c^2$

+8c

-3c

$(3c+2)(4c-1)$

12. $2x^2 - 5x + 4$



A	M
$-5x$	$+8x^2$

2, 4
1, 8

–

No factors!

Prime

Unit 8: Solving Quadratic Equations

To solve a quadratic equation, use one of the following methods:

- Factoring
- Quadratic Formula

1. $x^2 + 8x = 0$

$x(x+8)=0$

$x=0$ | $x+8=0$

$x=-8$

$x=\{0,-8\}$

2. $4x^2 = 10x$

$-10x \quad -10x$

$4x^2-10x=0$

$2x(2x-5)=0$

$2x=0$ | $2x-5=0$

$\frac{2x}{2}=\frac{0}{2}$ | $+5 \quad +5$

$x=0$ | $\frac{2x}{2}=\frac{5}{2}$

$x=\frac{5}{2}$

$x=\{\frac{5}{2},0\}$

3. $x^2 + 5x = 6$

$-6 \quad -6$

$x^2+5x-6=0$

$(x+6)(x-1)=0$

$x+6=0 \quad | \quad x-1=0$

$x=-6 \quad | \quad x=1$

$x=\{-6, 1\}$

4. $x^2 = 18x - 81$

$-18x+81 \quad -18x+81$

$x^2-18x+81=0$

$(x-9)(x-9)=0$

$x-9=0 \quad | \quad x-9=0$

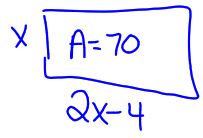
$x=9 \quad | \quad x=9$

$x=\{9\}$

5. The length of a rectangle is 4 inches less than twice its width. If the area of the rectangle is 70 square inches, what are its dimensions?

let x = width

let $2x-4$ = length



$A = lw$

$70 = x(2x-4)$

$70 = 2x^2 - 4x$

$-70 \quad -70$

$0 = 2x^2 - 4x - 70$

$0 = 2(x^2 - 2x - 35)$

$0 = 2(x+5)(x-7)$

$x+5=0 \quad | \quad x-7=0$

$x=-5 \quad | \quad x=7$

Reject

width = 7 in

length = $2x-4$

$2(7)-4$

10 in

Completing the Square

Rewrite each expression by completing the square

1. $a^2 - 4a + 15$

$(a^2 - 4a + 15 \boxed{-11}) \boxed{+11}$

$-4 \to (-2)^2 \to 4$

$(a^2 - 4a + 4) + 11$

$(a-2)^2 + 11$

2. $c^2 + 20c - 40$

$(c^2 + 20c - 40 \boxed{140}) \boxed{-140}$

$+100$

$\frac{20}{2} \to (10)^2 \to 100$

$(c+10)^2 - 140$

3. $x^2 + x + 1$

$(x^2 + 1x + 1 \boxed{-3/4}) \boxed{+3/4}$

$1/4$

$\frac{1}{2} \to (\frac{1}{2})^2 \to \frac{1}{4}$

$(x + 1/2)^2 + 3/4$

Solve for x by completing the square.

1. $a^2 - 4a - 15 = 0$

$+15 \quad +15$

$a^2 - 4a \boxed{+4} = +15 \boxed{+4}$

$\frac{-4}{2} \rightarrow (-2)^2 \rightarrow 4$

$\sqrt{(a-2)^2} = \sqrt{19}$

$a - 2 = \pm\sqrt{19}$

$+2 \quad +2$

$a = 2 \pm \sqrt{19}$

2. $n^2 - 2n - 15 = 0$

$+15 \quad +15$

$n^2 - 2n \boxed{+1} = 15 \boxed{+1}$

$\frac{-2}{2} \rightarrow (-1)^2 \rightarrow 1$

$\sqrt{(n-1)^2} = \sqrt{16}$

$n - 1 = \pm 4$

$n - 1 = 4 \qquad n - 1 = -4$

$+1 \quad +1 \qquad +1 \quad +1$

$n = 5 \qquad n = -3$

$n = \{-3, 5\}$

3. $r^2 + 4r + 3 = 0$

$-3 \quad -3$

$r^2 + 4r \boxed{+4} = -3 \boxed{+4}$

$\frac{4}{2}(2)^2 \rightarrow 4$

$\sqrt{(r+2)^2} = \sqrt{1}$

$r + 2 = \pm 1$

$r + 2 = 1 \quad r + 2 = -1$

$-2 \quad -2 \quad -2 \quad -2$

$r = -1 \quad r = -3$

$r = \{-3, -1\}$

4. $c^2 + 20c - 40 = 0$

$+40 \quad +40$

$c^2 + 20c + \boxed{100} = 40 + \boxed{100}$

$\frac{20}{2} \rightarrow (10)^2 \rightarrow 100$

$\sqrt{(c+10)^2} = \sqrt{140}$

$\sqrt{140}$

$\sqrt{4} \quad \sqrt{35}$

$2\sqrt{35}$

$c + 10 = \pm 2\sqrt{35}$

$-10 \quad -10$

$c = -10 \pm 2\sqrt{35}$

Unit 9: Rational Expressions and Equations

Simplify each expression or solve each equation for x.

1. $\frac{x^2 - 12x + 20}{x - 10}$

$\frac{(x-10)(x-2)}{(x-10)}$

$x-2$

2. $\frac{2n^2 - 10n}{n^2 - 9n + 20} \cdot \frac{n^2 - 8n + 16}{4n^2}$

$\frac{2n(n-5)}{(n-4)(n-5)} \cdot \frac{(n-4)(n-4)}{4n^2}$

1

$\frac{2n(n-4)}{4n^2}$

$\frac{n-4}{2n}$

3. $\frac{3y+9}{y+2} \div (y+3)$

$\frac{3y+9}{y+2} \div \frac{y+3}{1}$ ← mult. by the reciprocal!

$\frac{3y+9}{y+2} \cdot \frac{1}{y+3}$

$\frac{3(y+3)}{y+2} \cdot \frac{1}{y+3}$

$\boxed{\frac{3}{y+2}}$

4. $\frac{2x^2}{x^2-12x+20} - \frac{4x}{x^2-12x+20}$

$\frac{2x^2-4x}{x^2-12x+20}$

$\frac{2x(x-2)}{(x-2)(x-10)}$

$\boxed{\frac{2x}{x-10}}$

5. $\frac{x+24}{5x+20} + \frac{5x}{5(x+4)}$

Factor to find common denom.

$5(x+4)$

$\frac{x+24}{5(x+4)} + \frac{5x}{5(x+4)}$

$\frac{x+24+5x}{5(x+4)}$

$\frac{6x+24}{5(x+4)} \rightarrow \frac{6(x+4)}{5(x+4)} \rightarrow \boxed{\frac{6}{5}}$

6. $\frac{x-7}{6} = \frac{4}{x+3}$

$(x-7)(x+3)=24$

$x^2-7x+3x-21=24$

$x^2-4x-21=24$

$-24 \quad -24$

$x^2-4x-45=0$

$(x+5)(x-9)=0$

$x+5=0 \quad | \quad x-9=0$

$x=-5 \quad | \quad x=9$

$x=\{-5,9\}$

4. $\frac{2}{x-3} - \frac{1}{4x-12} = \frac{2}{x-5}$

$4(x-3)$

→ common denom.
Simplify
Cross mult.

$\frac{8}{4x-3} - \frac{1}{4(x-3)} = \frac{2}{x-5}$

$\frac{7}{4(x-3)} = \frac{2}{x-5}$

$7(x-5) = 2 \cdot 4(x-3)$

$7x-35 = 8(x-3)$

$7x-35 = 8x-24$

$-7x \qquad -7x$

$-35 = x-24$

$+24 \qquad +24$

$-11 = x$

$x = -11$